REVIEWS

Theory of Laminar Flow (vol. IV, *High Speed Aerodynamics and Jet Propulsion*). Edited by F. K. MOORE. Princeton University Press, 1964 869 pp. £10.

Introduction

The history of this volume makes it particularly difficult to review. The facts as stated in the editor's preface are that the volume had its beginnings under the joint editorship of Prof. Lester Lees and Prof. R. C. Lin and that much of this early work has been lost. The article on basic theory by Prof. Lagerstrom, written in 1953, has however been preserved whilst the remainder of the book under the editorship of Prof. F. K. Moore (appointed in 1957) was written in 1958–60. The publisher's imprint is 1964 but I did not receive my copy until late in 1965. Moore speaks of the vigorous development of various aspects of boundary-layer theory in the latter '50's and claims, not without justification, that as a consequence the authors were able to present 'rather well filled-in pictures of their subjects' whilst admitting that advances have in fact occurred in the period up to publication date. He adds that 'the requisite effort to add discussions of this new material has generally not been made'.

How does one assess such a work? It has a different value for the experienced research worker and teacher from that for a research student new to the subject who might be misled by parts which are out-of-date or incomplete. How does it stand in relation to *Laminar Boundary Layers*, for instance, which though not strictly comparable does contain much common material and which with an editor's preface dated 1961 and a publisher's imprint dated 1963 contains newer material?

My indirect answer to the first of these questions is that in my view the day of the composite encyclopaedic or semi-encyclopaedic work is over and I suppose I am entitled to say this as well as any one in view of my own experiences. In saying it however I am saying no more than that times have changed since Goldstein's *Modern Developments* appeared in 1938. No one would I think deny the success of these volumes (perhaps I can say this since my own role then was a small one) or dispute the prime importance of the role played by Goldstein through them in the development of the subject of fluid mechanics. This subject was then at a stage where drawing all the relevant threads together in a unified treatment was not only possible but desirable and gave a new impetus to research as well as setting into perspective the researches already undertaken. The subject has grown at such a rate and diversified in such a way that this unification is scarcely possible any longer—witness the sheer size of the task the editors of the Princeton series faced when they laid their plans in 1947/8 and consider how much the subject has grown since then. Moreover, the period between the time when the team of authors is chosen and the time of publication also seems to have grown inordinately. This is partly because the time in press appears, for reasons I do not claim to understand, to have increased and also because the time is governed by the time taken by the slowest author. This leads to the unfortunate and disheartening result that the speediest authors find their contributions more out-of-date (or more in need of revision) than their slower counterparts. Thus my solution is clear—no more composite works but monographs instead. These in their very nature will be shorter and more up-to-date when they appear and publishers are only too ready, one gathers, to compete for the privilege of publishing them. (In this connexion let me plead that the individual sections of this vol. IV of the Princeton series should be issued, as have some of the others, in paperback form.)

I will defer my answer to the second question until later in this review.

Contents

The volume under review consists of seven sections:

A. Introduction, by F. K. Moore.
B. Laminar Flow Theory, by P. A. Lagerstrom.
C. Three-dimensional Boundary Layers, by A. Mager.
D. Theory of Time-Dependent Flows, by N. Rott.
E. Hypersonic Boundary-Layer Theory, by F. K. Moore.
F. Laminar Flows with Body Forces, by S. Ostrach.
G. Stability of Laminar Flows, by S. F. Shen.

To take these in turn, A is brief—some seventeen pages—and apart from its obvious task of defining the scope of the work its main purpose seems to be to rectify some of the omissions in the succeeding pages, in particular it includes further remarks on separation, separated flows and approximate methods. In judging this one has to heed the comment in the preface: 'A separate article on separated flows had originally been planned. This did not materialise and therefore a rather abrupt discussion of separation and separated flows is included in Section A'.

The editor might also have added, but did not, that Sections B 14 and 16, C 18 and D 11 and 13 contain a few pages on this topic. One cannot help feeling, nevertheless, that it is a pity that the original plan was not adhered to and, on quite a different issue, that the editor did not allow himself more scope in his introduction.

Page 8 of Section A, which contains a discussion of the separation singularity, warrants comment. Moore starts by integrating, parallel to the boundary, the equation of motion in the boundary layer and states that equation (3–2) is a direct consequence of this integration. I have not been able to reproduce this equation except as an approximation, whereas I can find nothing in the text to suggest that it is anything but exact. In any event, except for the possible reason that its form is suggestive of the result required, it scarcely seems necessary to start here—the equation of motion itself suffices. In principle what is done does not seem to contain more than the argument given by

Stewartson (*Theory of Laminar Boundary Layers in Compressible Flow*, O.U.P., 1964, pp. 75, 76) who also gives references to earlier relevant work.

Differentiation of the equation of motion twice with respect to y and the use of the conditions $u = v = \partial u/\partial x = 0$ at the wall lead to the conclusion that

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)_0^2 = \nu\left(\frac{\partial^4 u}{\partial y^4}\right)_0, \tag{1}$$

where the suffix zero denotes values at the wall.

Thus either $(\partial^4 u/\partial y^4)_0$ vanishes at the separation point or it does not. If it does not then (1) coupled with the condition $(\partial u/\partial y)_0 = 0$ at $x = x_s$, the separation point, gives

$$(\partial u/\partial y)_0 \propto (x_s - x)^{\frac{1}{2}} \tag{2}$$

near separation.

Stewartson remarks that a consistent expansion for the stream function ψ near $x = x_s$, $y = 0$ can be worked out,

either (i) on the assumption that ψ is an analytic function of $x_s - x$ and y—which one may note implies $(\partial^4 u/\partial y^4)_0$ vanishes at separation;

or (ii) on the assumption that (2) holds—which implies that ψ is not an analytic function of $x_s - x$ and y. (The fact that a consistent expansion, compatible with particular numerical solutions, can be obtained in this case is validated by the work originated by Goldstein and followed up by Stewartson, Leigh, Terrill and others.)

This seems to be a more straightforward way of *surmising* the existence of the square root singularity than Moore's. In fairness I must add that Moore's account too makes it clear that it is no more than a surmise based on the non-vanishing of $(\partial^4 u/\partial y^4)_0$ at separation.

It is perhaps worth adding that another proof of the square root result appears in Landau & Lifschitz, p. 152, but this is based on the unproved assumption that, for given y, $x_s - x$ can be expanded as a regular series in $u - u_s$.

No one will I think question the value of Section B which takes up between a quarter and a third of the whole book. This more than any other section is the one we have been waiting for and is in my view a masterful account of the subject up to the time of its completion in 1956*—the latest cited references are three dated 1957, all of Caltech. origin. Fortunately much of this first completed section is so fundamental in nature that the passage of another ten years does not affect large parts of it. However, there is a residuum where significant developments have subsequently taken place. It would be a poor tribute to Lagerstrom, who for instance did much to excite interest in Oseen's equations and on the role of co-ordinate systems, if this were not so. Even on the fundamental side changes would, I suspect, have been made if it had been written later. For example the need for the assumption of isotropy has been given detailed examination by A. E. Green; he starts by remarking that rigid body rotations do not affect the stress (apart from its orientation) and then shows, by considering two motions which differ only by such a rotation, that if the

* There is a 3-year discrepancy between what Lagerstrom and Moore say about this date.

stress is a function of the velocity gradients only then that function must be an isotropic function of the rates-of-strain.

The theoretical part of Section B is as one might expect from its authorship heavily weighted towards exact solutions and in this context it is a little surprising that no results involving suction are included, particularly the rotating cylinder with suction and the asymptotic suction profile. The treatment of approximate methods tends to err on the side of brevity and rather surprisingly one has to look under compressible flows to find the discussion of approximate methods for incompressible flows.

Section C also contains much of permanent value but here again further developments have occurred. Maskell's work on separation which is discussed in detail provides a very solid foundation for this topic, but again this now has to be viewed against the background of the more recent work by Lighthill in *Laminar Boundary Layers*. Also Raetz's ideas, first formulated in his 1957 paper, find no mention; this seems to be an important omission in view of the light these ideas throw on the structure of the three-dimensional boundary-layer equations. It is only fair to add however that this paper was issued as a Northrop report and the author of Section C is not alone in overlooking its importance. Edge and corner effects are now more thoroughly understood as is the flow near a saddle point of attachment. None of these criticisms, save possibly the omission of Raetz's work, is a criticism of the presentation of the material available to Mager when he wrote his article, they simply serve to illustrate the difficulties of producing a composite work.

Section D is written by one who has made the subject of time-dependent flows particularly his own and his account is to be welcomed. As might be expected the chapter has much ground in common with J. T. Stuart's article in *Laminar Boundary Layers* on 'Unsteady Boundary Layers'. It is fascinating to compare the two, particularly since in addition to work already published each was also aware of work initiated by himself and his co-authors in progress at the time of writing. Rott plunges into his topic with little introduction whereas Stuart gives an important discussion of the role played by the basic parameters in determining the relative importance of the various terms in the equations. Stuart also in writing about acoustic streaming has the advantage of being able to include the results of one of his own investigations which was in fact not published until 1965. Rott, on the other hand, benefits in his account of periodic solutions by having at his disposal the results of his joint investigation with Lam. Rott's article goes beyond the scope of Stuart's in including for instance a useful discussion of the boundary layer behind a progressing shock on a flat plate.

Quite rightly in view of the nature of this volume and the existence of Schaaf's article in volume III, Moore in Section E limits his treatment of the hypersonic laminar boundary-layer to the case when the mean free path can be neglected in comparison with the boundary-layer thickness and the mechanics and thermodynamics of a continuum apply. Thus the main effects dealt with concern first of all 'real-gas complications due purely to high temperature—in this connexion we shall emphasize a binary reactive mixture of atoms and molecules with suitably varying vibrational specific heat—and second effects produced even in an ideal gas, by high Mach number...'. In his concluding remarks Moore makes it clear that he has not attempted to give a balanced review of the state of knowledge of hypersonic viscous flows but 'to give in more than usual detail the linearized "analogy" problems'. Here he is referring to the analogy problems of Couette and Rayleigh flows and claims that they are just as useful and suggestive in the hypersonic regime as in classical incompressible flow. One's views of the article must obviously be coloured by one's needs but for my part I find this approach both attractive and instructive.

In his preface Moore singles out two articles, viz. Sections B and F, for special mention, categorizing them as unique reviews. Section F—something less than a quarter of the volume—like Section B is in my view worthy of this special mention. The body forces discussed are in the main gravitational, centrifugal and Coriolis but there is a short section at the end—some twenty pages—on magnetohydrodynamics. To deal with this latter first it does not pretend to do other than present the fundamentals, to discuss some representative problems and to relate, where possible, its results 'to corresponding studies in gravitational convection, with a view to obtaining insights and techniques for new problems'. This is an interesting approach successful in its limited way but as one might expect has to lean heavily on referring the reader to other literature. As regards the main body of the article it might be useful to define its terminology; forced flows have their usual connotation but fluid motions which are due entirely to the action of gravity are called natural flows. Free convection is reserved for natural flows which are not constrained to a finite region by boundaries. This distinction between natural and free flows as the author admits is not universal. Ostrach deals in comprehensive fashion with problems of free convection, combined free and forced flows, natural convection and unsteady flows with body forces. He inserts a section on thermal instability early in his article dealing with it there in a qualitative fashion, reserving the mathematical treatment to his penultimate chapter and dealing there with both natural and free convection problems. There can in my mind be no question of the importance of this article as a whole, whether one's concern is with the details of a particular flow or with its stability. I have to confess that some of the work presented here was new to me and can only add that I found bringing myself up-to-date through Ostrach's article a pleasure.

Like Section D, Section G on stability has its counterpart in *Laminar Boundary Layers*, though the latter also covers some of the stability problems of Section F as well. The scope of Shen's article can be seen from the following quotation.

'Fundamental aspects of the theory of laminar stability have recently been summarized by Lin (*The Theory of Hydrodynamic Stability*, C.U.P., 1955). Without going deeply into the subtle mathematical questions we devote ourselves more to a general understanding of the subject and the various applications which have been made in problems of technical interest.' With Lin's book in the background this seems a natural approach. Chapters 1 and 2 give an account of parallel flow problems, particularly the plane Poiseuille and Blasius flows, though I must confess I find Section G13 on eigenvalues for the higher modes too condensed to be very informative. One has to dig here for instance to find out exactly what has been established for the plane Couette flow. The most useful parts I judge to be Chapters 3 and 4, the former dealing with various extensions and the latter with longitudinal vortex-type disturbances. Chapter 5 on behaviour subsequent to the onset of infinitesimal disturbances might have been written differently had it been written today, in view of the developments which have recently taken place. It is however, perhaps unfair to judge it on this account and anyway let me hasten to add that it does contain references which enable the reader to see for himself some of these developments.

Apart from the price my most serious criticism concerns the indexing and references. The index is a combined subject and author index. As regards subject it is by no means full enough—passages that one knows to exist may be difficult to trace—but it is on the reference side that the breakdown really occurs. Not all the authors of references are indexed and in a long chapter which may contain as many as 150 references, tracing the comments on a particular paper may be very tedious. *Laminar Boundary Layers* overcomes this difficulty very neatly by arranging the references alphabetically by author instead of in the order to which they are referred and then inserting after each reference the page number on which discussion of the reference may be found. Lest it be thought I have a vested interested in *Laminar Boundary Layers* in view of my many favourable references to it I ought once and for all to disclaim the slightest responsibility for it.

Conclusions

I believe that, in the hackneyed phrase, this is a book no one with interests in laminar flow can fail, if he can afford it, to put on his shelves for frequent reference. To answer the second question I posed in the Introduction, my own personal preference where there is an overlap in content and outlook with *Laminar Boundary Layers* is for the latter. To judge by my own experiences it is easier to find one's way around *Laminar Boundary Layers* than it is to do so in the volume under review and I do not think my familiarity with the former has unduly influenced me in saying this. I also find in *Laminar Boundary Layers* the introductory material to its various topics more helpful and the articles themselves certainly not less so.

Without wishing to denigrate the rest of the volume I agree with Moore in singling out for special mention the articles by Lagerstrom and Ostrach. Though the content of the former overlaps parts of *Laminar Boundary Layers* its outlook is different and exceedingly useful and revealing. Ostrach's contribution has less overlap with *Laminary Boundary Layers* and is a most helpful unification of the subject, more up-to-date than some of the other articles since the author was able to draw on his own unpublished work.

The volume as a whole maintains a standard compatible with that expected by the original editors of the series of twelve when their plans were first laid and it is good to see their imaginative project brought to fruition in this way. The time involved has no doubt been much longer than they expected and sets into perspective Sears's achievement in getting vol. VI of the series out as quickly as he did.

Finally I reiterate my suggestion that for the future authors should think in terms of monographs rather than collected works and couple with this the plea for reproduction of individual sections (particularly B and F) in paperback form. At more than twice the price of *Laminar Boundary Layers* it really is expensive and in danger, particularly for research students and indeed for all but senior staff, of pricing itself out of the market.

L. HOWARTH

Elements of Hypersonic Aerodynamics. By R. N. COX and L. F. CRABTREE. English Universities Press, 1965. 243 pp. 35*s*.

The problems that arise in considering gas flows at Mach numbers greater than about 5 are sufficiently distinct from those for supersonic flows at lower Mach numbers to justify separate treatment, and the term 'hypersonic flows' is used to describe them. Although of comparatively recent origin, the volume of research on hypersonic flows has grown very rapidly. This is due in part to the stimulus of practical interest in the re-entry of ballistic missiles and space craft to the atmosphere, and in future transport vehicles flying with very long ranges within the atmosphere. It is also partly due to the fundamental interest of many new problems of fluid motion, chemical kinetics and gas-surface interactions that arise. All students of aerodynamics require some knowledge of hypersonic flows if only to place their work on supersonic flow in proper perspective, and the newcomer to research or design related to hypersonic vehicles needs guidance when confronted with the extensive and scattered literature.

For such reasons books and review articles are important, and it is fortunate that excellent books like those by Hayes and Probstein, Chernyi, Dorrance, and Clarke and McChesney have been written early in the development of the subject. Knowledge is, however, increasing rapidly, and the present new book is to be welcomed especially because it discusses not only problems that would arise if the fluid behaved as a perfect gas, but also those arising from the effects of viscosity and high-temperature or low-density gas behaviour.

Following a brief introduction, the book is divided into two parts: Dr Cox being mainly responsible for the first and Dr Crabtree for the second. Part I is concerned with the continuum flow of an inviscid non-heat-conducting perfect gas, and contains chapters on general flow relationships, similarity principles, and on the flow past slender, blunt, and blunt-nosed slender bodies. Part II begins with chapters on high-temperature real-gas effects in inviscid flow, and on hypersonic boundary layers including the case where high-temperature effects are present. There is then a brief chapter on the interaction of the boundary layer with the external flow, and finally a chapter on supersonic and hypersonic flow at low density.

There is little or no discussion of such topics as experimental methods, propulsion problems, ionization effects, or magnetogasdynamics. Even so, a very wide range of material is included, and the authors are to be congratulated for condensing this into a volume of some 240 pages. An inevitable consequence is that equations and results are often quoted without detailed derivation so that the reader must consult other books or original papers for further details, or rely to an unusual extent on the accuracy of the text. Reference to the literature is aided by a list of nearly 300 references and the long errata sheet corrects all factual errors noted by the reviewer at a first reading.

In the preface the authors state that the manuscript was completed in 1963, and progress has been so rapid that they have been unable to include several refinements to the theory and other important developments. Thus, for example, the discussion of hypersonic vehicles is limited to ballistic and boost-glide trajectories, and there is little reference to problems arising for steady hypersonic flight within the atmosphere, or of associated developments in the concept of flight corridors. There is also little discussion of lifting hypersonic vehicles such as caret wings based on simple shock systems.

The reader will, however, have little difficulty in filling such gaps by reference to recent articles, and the book can be recommended as a concise and readable introduction to the theory and physical nature of the problems arising in hypersonic flow. Although it is particularly suitable for postgraduate students or final-year undergraduates specializing in aerodynamics, it will not doubt also prove useful for reference purposes to more advanced readers.

D. W. HOLDER